jects was increased, and hence the size of, and workload for, each examination were reduced. Andrew Wilson said that this approach had been adopted by the Society of Actuaries but had not resulted in improved pass rates, possibly because students didn't strive for so much.

Another speaker felt that the proposal to limit the student to four attempts at subjects A, B, C and D might provide increased incentive, but would prove unworkable, because of the number of special dispensations for deserving cases.

In summary, Chris Daykin emphasised that completion of the examinations produces an embryo of an actuary, to be developed further by learning through work experience and by CPE. The new system of objective-based learning and loose-leaf tuition material will make it easy to modify the syllabus and tuition material, as required, a little at a time; thus removing the need for long periods between major revisions.

Malcolm Lee

Measure for Measure

3. Chiron and the Joseph Effect

Nick Ryan & Jon Spain continue their series on performance measurement with some thoughts on simulation.

The concept of Discounted Value Return (DVR) given in the previous article has a theoretical and empirical background to its development, which is explored in this and the next article. This article begins with consideration of a critique of Market Values (MV) and their associated Time Weighted Return (TWR). We begin at some distance from our subject beyond the orbit of Saturn.

We begin beyond the orbit of saturn

Planetary Disturbances

Hahn and Bailey[1] have investigated the dynamics of the anomalous planetoid 2060 Chiron. This object has an orbit which lies near that of Saturn, and it is also strongly influenced by Jupiter. The interest which their paper holds is twofold, the statistical methodology, and the chaotic conclusion.

They take four of the principal orbital elements; semi-major axis, eccentricity, inclination and longitude of ascending node. The semi-major axis is known to 10^{-4} Astronomical Unit, the eccentricity to 10^{-5}, while the inclination and the longitude of ascending node are known to 10^{-4} degree. Hahn and Bailey consider 81 different versions of the orbit of Chiron, consisting of all possible variants of the four elements taking each one in turn at its current central value, and at + or - 1 deviation. Each version was followed on a computer model, using the known laws of gravitation.

As with most small objects (Chiron has about 10,000 times the mass of Halley's Comet) near large bodies (Jupiter and Saturn are respectively 318 and 95 times the mass of the earth), Chiron's orbit exhibits resonances with its massive neighbours; that is, its orbital motion tends to bear a simple ratio, such as 2:3 or 3:5, to that of its giant partners.

What is surprising is the way in which Chiron behaves, when allowed to evolve over timescales which are moderately large multiples of its natural period (which is of the order of 50 years). As Hahn and Bailey say: *"The difference in this evolution from that expected on a 'random walk' hypothesis is remarkable, with variations in semi-major axis frequently occurring as discrete 'jumps' from one near-resonant state to another."*

do our numerical values have better accuracy than a few tenths of one percent!

For three centuries Newtonian mechanics has been the paradigm of the exact sciences; and in the case of Chiron the key numbers are known to a relative accuracy of one part in a hundred thousand. How much more circumspect should we be in drawing conclusions in a discipline which is statistical to start with, and where rarely do our numerical values have better accuracy than a few tenths of one percent!

The Joseph Effect

Hydrologists were the original discoverers of the so-called "Joseph Effect". In Genesis Chapter 41 Joseph interprets Pharaoh's dream of the seven fat kine and the seven lean kine. This symbolised the consequences of fluctuations in the annual inundation of the Nile. In more recent times these were studied by H E Hurst (Abu Nil), among others, in connexion with the Low and High Dams at Aswan. The effect, of course, is that several "good" years will be suddenly followed by several "bad" ones - and *vice versa*. Benoit Mandelbrot[2] remarks:

"*It has become prudent to expect every branch of Physics to include its own instances of Joseph Effect. The more successful branches no longer seem characterised by their success in dealing with it, but merely by the fact that they have the choice of centering their efforts on Joseph-free areas.*"

Examples today include the persistent droughts in the Horn of Africa, and the possibility of spurious signals about Global Warming. Another is the orbital behaviour of Chiron. And then there is Performance Measurement.

Simulating Economics

If economics is treated as a science, then it is fair to suppose that there are no Joseph-free areas; experience of real funds in real markets casts doubt about the validity of TWR statistics and the MV oriented approach to Performance Measurement. The DVR method arose out of dissatisfaction with these methods.

Simulation methods can test hypotheses about the usability, robustness and statistical significance of different systems. This approach uses three main groups of programs:

1. A randomiser to produce "good quality" Gaussian variables for market-simulation purposes.
2. A fund model that permits realistic simulation of fund behaviour.
3. A powerful algebraic solution to the Internal Return Problem, enabling, in this context, rapid calculation of the TWR.

Pseudo-Random Numbers

Pseudo-random number generators are commonly used in computer applications; however, care is needed on two fronts.

We need repeatability, in order to rerun experiments, for instance simulating the same fund in different markets, or simulating different funds under the same market conditions[3]. Equally important is the reproducibility of the results by those wishing to check, and thereby convince themselves of the validity of the conclusions.

On the other hand, we don't want built-in periodicities or high spectral correlation[4]. This is particularly vital when dealing, as we are, with Time Series Analysis, wishing to avoid spurious secular (time domain) or spectral (frequency domain) signals. Such resonances are precisely what we are trying to detect, and it is fatal to feed them in as artefacts[5].

So it is sensible to use a special pseudo-random generator, rather than the built-in one that comes with most computer systems. Its output is a pseudo-random number with a rectangular distribution in the range [0,1). This is then converted to a Gaussian abscissa, from a Standard Normal Distribution, giving the positive or negative number of standard deviations away from the mean.

Fund Behaviour

The distribution of market returns is expected to be Gaussian[6,7], but the process is not stationary: the volatility has long-period secular changes as well as periods related to shorter term stock market cycles. The sampling grid is thus important. And because we have the basic relationship,

$$\text{Volatility} = k\sqrt{(\text{Time})},$$

the inter-sample period can affect the results.

Individual funds vary, not only one from another, but also over time. Their relative income and outgo will change, probably at different rates; crudely these are the sum of inflationary and demographic factors. Therefore they can be varied in the model fund, relative to each other and relative to fund size. In addition, the model fund can be assumed to have a long-term return, R, which for now it can be taken to be the rate assumed actuarially for funding purposes.

The Rate of Return

The final element is the algorithm to calculate the TWR. Lake's Algorithm[8] has been in use for real funds for over 10 years. A key result is that in a "pure" fund, growing smoothly with time, TWR = R. In a real fund, then, the algorithm tells us what the achieved return is; in a model any discrepancy between the postulated return, R, and the calculated TWR is a signal that something needs investigating.

Developing the procedure as a whole we find the following results:

1. Even though R is given, calculating it via TWR can take a long time (c. five years in simulated time) to converge.
2. Short-term market shocks affect the results - that is, TWR can be quite inaccurate in short period calculations.
3. Markets and funds exhibit "bounce-back", that is, several good results are followed by bad, or vice versa. The technical term is "anti-persistence".
4. Use of a Monte Carlo method enables significance to be calculated. Results suggest that for high significance some 50-

60 sampling points are needed; for example, where quarterly data are used, some 12-15 years.

Conclusions

The overall conclusions are:

- Results based on the short term are untrustworthy; our old friend the Joseph effect is at work;
- Random Walk theory is knocked on the head, because pure Brownian motion does not exhibit anti-persistence; markets appear rather to behave like "1/f noise"[9];
- MV based measures are unreliable as input to the monitoring process, because a single freak value, though ultimately transient, can produce long lasting perturbations; and
- TWR results are inherently unstable.

Nick Ryan
Jon Spain

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Staple Inn Lawyer

After the battering that final-salary occupational pension plans have had over the last five years (earnings capping, lump sum capping, index linking, bribes in favour of personal pensions *et al*) one might be forgiven for thinking that the present government has an animus against these animals. Add to that the imprecations of Christopher Fildes in *The Spectator* and Neil Collins in *The Telegraph*, and the conspiracy paranoia matches that of Alan Bleasdale in GBH. If I were an actuary, I might be tempted to think of re-qualifying as an accountant.

Do final salary schemes have a future? The bad news is that the age of the very large employer (the steel industry, the nationalised utilities, coal, motor manufacturers) is coming to an end – and for these employers, final salary arrangements held a peculiar fascination. And the costs of final salary arrangements, in real terms, must be rising as the effects of the legislation begin to bite. But, on the other hand, the recent report from Noble Lowndes seems to suggest that there is no panic just yet amongst employers and that the predicted rush to money-purchase has yet to show itself in the figures. And, more encouragingly, a recent White Paper issued by the French Government indicates that as the demographic profile in France slowly changes to the detriment of repartition schemes, the saviour will be the final salary occupational scheme, albeit on an industry-wide basis. Similar conclusions have come out of a variety of European conferences held by the International Labour Organisation, the Council of Europe and the European Institute of Social Security over the last 12 months. Perhaps just as the UK moves to diminish the role of pension schemes (even the Labour Party is now threatening to limit tax relief to basic rates) the rest of the world is moving the other way. It would not be the first time that a Great British Invention was taken over by overseas competitors and improved. The only hope is that they may re-export it back to us. Now that the European Commision has just published the text of its draft of a proposed directive controlling pension funds, the rot may stop.

The Barber case never stops. The latest news (at the time of writing) is that a German case and a Dutch case may beat Coloroll, the British case, to the European Court of Justice – latest indications are that the first of the cases, devoted to the proper interpretation of Barber, will be heard some time in the Autumn. The problem is that the German case concerns book-reserve schemes and the Dutch case the industry-wide scheme for the window-cleaners (the charmingly named *Glassenwassers*), neither of which truly reflects the role of an occupational scheme in the UK. The problem with the dreaded Article 119 is that it applies to "pay" – and therefore to the employment relationship. Pension