Measure for Measure

4. The Theory of Froth

Nick Ryan and Jon Spain compare DVR with MV and TWR.

In Part 2 of this series (*The Actuary*, June 1991) we considered the Discounted Value Return (DVR), and its consistency, as opposed to that of the Market Value basis and its related Time Weighted Return (MV and TWR). They are all competing methods of estimating the long run, *maintainable*, return. Then again, the fund simulation model described in Part 3 (*The Actuary*, October 1991) contains a quantity R, which, we can now reveal, is precisely this underlying rate. In the model fund, of course, we *knew* what R was, because we put it there. In real funds we don't know R; indeed, it is what the whole process of performance measurement is an attempt to discover.

Here we consider some theoretical background for how we would expect R, and its estimators, to behave, and some empirical evidence for our approach. We give more results on the relative long-run consistency of DVR as opposed to MV/TWR.

The Random Hit Ratio

Imagine that m hunters pursue n deer; every hunter is armed with a single-shot gun and has perfect aim, but because the hunters cannot talk to one another - perhaps it would alarm the prey - each selects a target at random. How many deer would you expect to be hit, and how many to remain unscathed?

When m = n, the problem becomes the so-called Derangement Problem[1], or Mad Secretary Syndrome. A secretary with n letters and n envelopes, matches them randomly. What are the chances that *every* letter will be in the wrong envelope? The number of such derangements, called D_n, has an exact algebraic formula,

$$D_n = n!\,(1-[1/1!] + [1/2!] - [1/3!] +\ldots+ (-1)^n\,[1/n!])$$

which is finite, but unfortunately cumbersome to calculate for more than small values of n. On the other hand we have the good luck, that it is very rapidly approximated by the analytic expression,

$$\lim D_n = n!/e$$

Leaving the details of a finite number of hunters and deer for actuarial winter evenings, the asymptotic proportion of deer hit is given by:

$$\lim r_{m,n} = 1-e^{-1} \approx .6321$$
$$(\text{as } m, n \rightarrow \infty)$$

This number is known as the *Random Hit Ratio*

Our Hunting Fathers

The stock market is a collection of independent hunters. They also have perfect aim; a fund manager who sets out to buy, for instance, Marks & Spencer, doesn't accidentally hit Glaxo. Of course, the single shot premise is artificial, and so is the underlying assumption that the probability of target selection is identical for all deer. But the simplified version will give us a starting point for our investigations.

On theoretical grounds, then, we might expect some 63 percent of achieved return to be sustainable in the future.

The rest is *froth*.

Froth is not necessarily a bad thing. It exists to provide the markets with elbow room. An ecological parallel is the function of the hedgerow. Apparently unproductive, it protects the productive area from erosion. Chopping down one hedgerow is comparatively innocuous; cut them all and see your topsoil blow into the next county; let all farmers cut them and create a dustbowl. The apparently unproductive part is a precondition for the productivity of the whole. From the viewpoint of a single fund, froth results from market fluctuations which happen to coincide in time with sampling points superimposed on the fund's maturation cycle.

DVR is very good at detecting froth. A survey of 27 real funds covering the six years 1979 - 1984 has been published[2]. The overall market values, representing about ½ percent of discretionary pension funds, increased from £192.4 million to £870.7 million, with net cash flow of £131.0 million during the review period. The 27 funds were subjected to 27 distinct treatments, by varying three parameters in three ways each (reminiscent of the astronomical studies of planetoid 2060 Chiron referred to in Part 3.):

1. Growth rates were taken at 50 ± 0 or 5 % of return.

2. Equities were valued at perpetuity or 20 years sale, and the 20 year valuations were taken at full income and price growth throughout or full income growth and half rate price growth.

3. Fixed interest securities were valued as notionally reinvested into equities, as reinvested into the 25 year High Coupon Gilt Index (HCGI), or as into 15 year HCGI.

The *summary* listing occupies some 22 pages of computer output, including two formats, showing on the one hand Between-Funds/

Within-Treatments, and on the other Between-Treatments/ Within-Funds; but to summarise the summary, this empirical study showed froth factors in the range 50 to 75 percent, with most in the 60 to 65 percent bracket.

If we think in terms of *expected* sustainable return, then froth is a deviation. It may not be possible or desirable to eliminate it - let us not cut down the hedgerows - but we do want to quantify it. We then have to examine possible reasons for any substantial divergence from *both* the theoretical expectation *and* the averages of the market and of the measurement universe. Even if precise reasons are difficult to detect, we should beware of anti-persistence, which we encountered in Part 3; very high recent returns may well be less likely in future. As John Allen Paulos remarks[3]:

"*I've always suspected that notions like a 'hot hand' or a 'clutch hitter' or a 'team that always comes back' were exaggerations used by sportswriters and sportscasters just to have something to talk about. There is surely something to these terms, but too often they're the result of minds intent on discovering meaning where there is only probability.*"

So the other vital part of DVR is the measure of sustainability.

Prudence and Intuition

How much is it prudent to assume? Here we have an interesting point of contact with actuarial intuition. Most studies suggest that the real return of the stock market, using the FT-Actuaries All Share Index as proxy, over periods of the order of 25 years, is some 5%.

One very simple calculation, which can be done almost on the back of an envelope, or at least with a pocket calculator, depends on the fact that the All Share Index and the Retail Price Index both started in early 1962. There was virtually no movement in the RPI for the first three months (it began in January), so we can treat it as roughly synchronous with the ASI, which began in April. For cosmetic reasons the RPI has been rescaled occasionally (though not the ASI - after all we *want* the market index to look big), and the latest (January 1987) chaining factor is 7.5665, or say 7.5 for approximate purposes.

(1) Calculate ASI/(7.5 x RPI);

(2) Take the nth root, where n is the number of years (and months taken as twelfths) since the start;

(3) Add the running yield.

It's the go for computational packages to be acronymously slightly off-colour, so we might call this one Simple Minded Attempt to Refocillate The Actuarial Stock Statistics (SMARTASS).

The result is usually about 5%. For instance, the May 1991 RPI, published in mid June, stood at 133.5; the FTA-ASI on 14 June was 1211.83, with a running yield of 4.86. SMARTASS produces the numbers:

1211.83/(7.5 x 133.5) = 1.210317

(1.210317)^(1/(29+2/12)) = 1.006566

100x(1.006566 - 1) + 4.86 = 5.52%

More sophisticated calculations employ refined versions of essentially the same method. For instance, Clay & Partners publish various reinvested indices[4]. Up to 1985 the numbers involve adding back $\frac{1}{12}$ of the running yield each month, and from 1986, the notional accrued dividend.

From, say, December 1963 to December 1990, the RPI increased from 13.8 to 129.9, while the FTA-ASI increased from 108.54 to 1032.25; the factors were quite similar, being 9.41 and 9.51, with a running yield of 5.5%: the result is a total return of 5.54%. Using the (as calculated by Clay & Partners) *reinvested* FTA-ASI, which increased from 108.54 to 4225.2, we have a return of 5.40%.

Taking figures from Bacon & Woodrow[5] which are somewhat differently presented, we have for the 24 years to 1989, annualised return on Equities = 16.03%, and annualised increase in RPI = 9.00%, giving a real return of 6.45 percent per annum.

So SMARTASS looks quite good, at any rate for ballpark purposes. Other sources may produce slightly different figures: a consequence, largely, of the way the numbers are compiled. The results are usually 5 to 6 percent real yield.

Yet most actuaries would hesitate to value a long term fund on 5 percent real yield. A typical pension fund figure would be 3 percent real with respect to prices, and somewhat less with respect to wages which over time tend to outstrip prices by between 1 and 2 percent per annum.

So actuaries intuitively expect *maintainable* returns, usable as a basis for the rational planning of the future, to be three fifths or thereabouts of the historic real yield.

Our results, both theoretical and empirical, suggest that this actuarial intuition is justified.

Correlations

One important outcome of DVR is the correlation between periods. Do the last five years tell us anything useful about the next five? If five years aren't helpful, what about ten? A central problem of Time Series Analysis is the tradeoff between significance and trend. Too few data points will not tell you anything statistically significant, but too long a scale can obscure underlying changes.

In a recent study[6], Wales remarks:

"*Determining whether time series of data from dynamical systems exhibit regular, stochastic or chaotic behaviour is a goal in a variety of problems. For sparse time series (those containing only of the order of 1000 data points), the goal may simply be to discover* *whether the series are chaotic or not.*"

We note that a *sparse* fund measurement based on quarterly data would take 250 years, which is a bit too long term even for actuaries.

In a practical sense, however, this does not matter, because over such a time scale not only the fundamentals of the fund, but also of the markets and indeed of the world economy will almost certainly change out of recognition. We need shorter range correlations, always keeping one eye open for the Joseph Effect (see Part 3, *The Actuary*, October 1991).

DVR to the Rescue

Fortunately, DVR methodology comes to our rescue. In Part 2 we gave calculations of DVR (for illustrative purposes as a series of annual results) *versus* MVR for a notional fund. This was constructed using figures published by PDFM[7] for private sector funds to obtain the proportions of Real/Monetary assets. There were four versions, depending on whether the two asset types were held to Perpetuity or assumed to be redeemed at 15 years, which we refer to as Variants A, B, C and D.

We now show the results of the corresponding calculations for a series of nineteen 10-year periods. In each period we ask whether the averaged "interior" five years were good predictors of the full ten years. These are compared in each case for DVR and MVR, and the better one starred, to give a "box score".

Table 1 below gives the complete outcome for the Perpetuity/Perpetuity model (Variant A). To give the full figures for every possible combination would consume too much space for this article; so in Tables 2 and 3 on the next page we summarise. Table 2 is in effect the right-hand margin and Table 3 the bottom margin of an expanded Table 1. Thus Table 2 shows the MVR performance of five years *versus* 10, together with the corresponding numbers for the four DVR variants. Table 3 shows the column means and deviations and the grand averages for MVR against each of the four DVRs.

What each computation shows is:

1. The outcome of the method (MVR or DVR Variant X, where X is A, B, C, or D) for the full 10 years.
2. The mean and standard deviation of the same calculation for the six possible sets of five consecutive years within the full 10.
3. The ratio of 5 years:10 years for the two contrasted treatments - the nearer to 1 this ratio is the better the method is at predicting itself.
4. The box score, showing whether M (=MVR) or D (=DVR) wins.

The general conclusions are:

- For individual five-year or 10-year columns DVR has a much lower relative standard deviation than MVR; it is internally more trustworthy.
- For the averaged ratio (relative performance in long run usage) MVR has greater standard deviation than its mean; the "worst" DVR (Variant D) has a mean more than eight times its standard deviation.
- For individual 10-year periods (relative performance as a

DVR TABLE 1

VARIANT A
Real = Perpetuity
Monetary = Perpetuity

10 Year Period Ended	MVR10	MVR5 Means	MVR5 SDevs	DVR10	DVR5 Means	DVR5 SDevs	MVR5/10	DVR5/10	Score M	Score D
Dec 72	9.17	9.48	2.78	9.40	8.54	.87	1.0338	.9085	*	
Dec 73	5.39	8.72	3.90	9.36	8.53	.87	1.6178	.9113		*
Dec 74	.84	6.15	6.83	9.43	8.84	1.34	7.3214	.9374		*
Dec 75	7.06	5.96	6.78	10.36	9.72	1.95	.8442	.9380		*
Dec 76	7.61	5.09	6.96	11.77	11.13	2.79	.6689	.9456		*
Dec 77	9.92	4.05	5.92	13.31	12.67	2.96	.4083	.9519		*
Dec 78	8.05	4.48	6.54	13.95	14.12	3.16	.5565	1.0122		*
Dec 79	9.70	8.93	10.89	15.68	15.82	2.84	.9206	1.0089		*
Dec 80	12.74	13.04	8.78	16.35	17.19	2.23	1.0235	1.0514	*	
Dec 81	10.00	15.01	8.63	16.87	17.95	1.27	1.5010	1.0640		*
Dec 82	12.47	17.64	6.16	16.68	18.03	1.17	1.4146	1.0809		*
Dec 83	17.94	19.89	4.38	16.93	17.83	1.42	1.1087	1.0532		*
Dec 84	26.65	21.59	3.96	17.38	17.33	1.71	.8101	.9971		*
Dec 85	20.31	20.55	2.56	17.25	16.56	1.60	1.0118	.9600	*	
Dec 86	22.16	21.70	2.81	16.80	15.96	.92	.9792	.9500	*	
Dec 87	18.67	21.84	2.71	16.44	15.74	.59	1.1698	.9574		*
Dec 88	19.31	21.84	2.70	16.59	15.92	.89	1.1310	.9596		*
Dec 89	21.46	21.31	3.03	16.21	16.19	1.02	.9930	.9988		*
Dec 90	17.51	19.33	3.90	16.02	16.54	.96	1.1039	1.0325		*
Col Mean	13.52	14.03	5.28	14.57	14.45	1.61	1.3483	.9852		
Col SDev	6.67	6.82	2.40	2.90	3.33	.79	1.4374	.0513		
Score M-D									4	15

DVR TABLE 2

Real Monetary		Variant A: Perpetuity, Perpetuity			Variant B: Perpetuity, 15 Years			Variant C: 15 Years, Perpetuity			Variant D: 15 Years, 15 Years		
10 Year Period Ended	MVR5/10	DVR5/10	Score M	Score D	DVR5/10	Score M	Score D	DVR5/10	Score M	Score D	DVR5/10	Score M	Score D
Dec 72	1.0338	.9085	*		.8987	*		.9666		*	.9618	*	
Dec 73	1.6178	.9113		*	.9215		*	1.0854		*	1.1241		*
Dec 74	7.3214	.9374		*	.9408		*	1.1451		*	1.2153		*
Dec 75	.8442	.9380		*	.9407		*	.8828		*	.8732		*
Dec 76	.6689	.9456		*	.9474		*	.8362		*	.8199		*
Dec 77	.4083	.9519		*	.9344		*	.7928		*	.7581		*
Dec 78	.5565	1.0122		*	1.0007		*	.8955		*	.8683		*
Dec 79	.9206	1.0089		*	1.0033		*	.9375		*	.9209		*
Dec 80	1.0235	1.0514	*		1.0471	*		1.0146		*	1.0065		*
Dec 81	1.5010	1.0640		*	1.0791		*	1.1020		*	1.1171		*
Dec 82	1.4146	1.0809		*	1.0865		*	1.1426		*	1.1476		*
Dec 83	1.1087	1.0532		*	1.0525		*	1.0588		*	1.0582		*
Dec 84	.8101	.9971		*	.9943		*	.9562		*	.9539		*
Dec 85	1.0118	.9600	*		.9619	*		.9738	*		.9744	*	
Dec 86	.9792	.9500	*		.9533	*		.9578	*		.9591	*	
Dec 87	1.1698	.9574		*	.9677		*	1.0029		*	1.0094		*
Dec 88	1.1310	.9596		*	.9663		*	1.0017		*	1.0058		*
Dec 89	.9930	.9988		*	.9994		*	.9989		*	.9994		*
Dec 90	1.1039	1.0325		*	1.0318		*	1.0548		*	1.0536		*
Col Mean	1.3483	.9852			.9856			.9698			.9909		
Col SDev	1.4374	.0513			.0524			.0939			.1121		
Score M-D			4	15		4	15		2	17		3	16

medium range predictor) MVR scores in 2 to 4 of the 19 sample periods; DVR scores 15 to 17 times.

The crude scoreline is that DVR beats MVR by about 5 to 1.

This accords with the result of our Monte Carlo simulations discussed in Part 3, but its external importance should not be overlooked. It does mean that the results of the last five years, when using MV/TWR, tell us very little about the *next* five. So when a fund fires its investment manager on the basis of past performance, it may be legitimate, in the sense that the last five years really were lousy, but it is no guarantee at all that the *same* manager might not have performed better in future. But it is unlikely to be legitimate to use MVR even for the past five years, because its variance is too large. DVR gives us much greater confidence that it is telling us something useful about the future, as well as about the past.

Looked at in another light, this is statistical confirmation that short-term-ism (ignoring whether it's good or bad for the economy) is actually *deleterious* to funds. The oft-repeated assertion that "*the long run is nothing but a succession of short runs*" is simply *not true*. It should be noted that we are *not* saying "*deleterious to funds collectively*" with the concealed implication that a single, as it were parasitic, fund can make a killing by breaking ranks – the deleteriousness applies at the level of the individual fund.

Conclusions

Thus the succession-of-short-terms approach, sometimes called Keynesian optimisation, but (since

DVR TABLE 3

	MVR10	MVR5 Means	MVR5 SDevs	DVR10	DVR5 Means	DVR5 SDevs	MVR5/10	DVR5/10	Score M	Score D
Variant A										
Col Mean	13.52	14.03	5.28	14.57	14.45	1.61	1.347	.985	4	15
Col SDev	6.67	6.82	2.40	2.90	3.33	.79	1.437	.051		
Variant B										
Col Mean	13.52	14.03	5.28	14.28	14.18	1.61	1.347	.986	4	15
Col SDev	6.67	6.82	2.40	3.24	3.64	.88	1.437	.052		
Variant C										
Col Mean	13.52	14.03	5.28	14.49	14.36	2.04	1.347	.990	2	17
Col SDev	6.67	6.82	2.40	3.34	3.66	1.36	1.437	.094		
Variant D										
Col Mean	13.52	14.03	5.28	14.16	14.03	2.18	1.347	.991	3	16
Col SDev	6.67	6.82	2.40	3.72	4.03	1.51	1.437	.112		

that derives from something Keynes didn't actually say) more appropriately called *iterative* optimisation, needs to be replaced, for the health of funds, individually and collectively, as much as that of the economy. The replacement is known, following Bellman[8], as Dynamic optimisation, though a better term would be *recursive* optimisation. DVR gives us the means to discriminate between the effects of the two approaches.

In our statistical and theoretical background some *caveats* are needed. Among the questions that arise is why certain funds show so much divergence from the Random Hit Ratio. One reason is that in actual funds we do not know the true underlying value of R and have to estimate it. We now know DVR to be a better estimator than traditional methods. We still expect there to be froth, for the same reason that farming needs hedgerows, but the *amount* of elbow room does not have to be analytically exact. Alternatively, some hunters may have a greater supply of bullets than their fellows, or a faster rate of fire. And some deer are larger or juicier than others.

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1848 And All That

Chris Lewin concludes the summary (which commenced in the March issue) of the rules and regulations governing insurance policies issued at the Assurance Office in the Royal Exchange, London, around 1660.

67. *If damage happen to any goods by fault of ship or by extremity of weather.*

68. *If any assurance be made of one special commodity and the same receive damage, the assured having other goods.*

69. *Upon arrest, restraint, the average shall be adjudged by the Judges.*

70. *If by shipwreck goods be utterly lost, or all or part saved. Item, if any ship or ships of whatsoever quality or making they be of should happen to be utterly lost, or have made shipwreck, in which there was merchandize assured, and if the same merchandize should happen to be utterly lost, the assurer shall be bound to pay his just sum asured, but after shipwreck the merchandize should be found floating or cast up upon the shore, and the asured will not make renunciation to the assurer, but will have the merchandize himself, by reason that he is further interested than the assurers are, then the assurers shall be bound to pay the charges in taking them out of the sea, and all other charges in washing, dressing, trimming and keeping of them, and al other chagres in recovering and saving of the merchandize, which charges shall be borne by way of average, as aforesaid, and in the average, the goods or merchandize shall be rated with all charges as they cost before the misfortune happend; And if the assured will claim the goods, whereafter by wetting, washing or drying the goods or merchandize should become worse in goodness or should want of weight, the impairing any goods or wants of weight after it is in the assured's possession, by his own desire, shall not charge the assurer, for the assured might have made renunciation and would not.*

71. *If part of the goods be well conditioned and part wet.*

72. *Upon all averages the commodities to be rated as they cost with charges.*

73. ***Upon demand of gross average, and the ship will not discharge till the same be decided.***

74. *If a ship willingly be run ashore for saving their lives, and lost or saved, or the goods saved, or the goods saved.*

75. ***Goods not stowed under hatches shall not charge the assurer in average nor otherwise.***

76. *If a ship strike upon any bar, rock, sand.*

77. *If divers men lade in one ship one commodity, and one man's taken away.*

78. *Leakage goods staved, or other goods hurt by opening hatches to cast overboard.*

79. *If by the fight of a ship, goods or merchandize receive hurt.*

80. *Of explaining of a gross average.*

81. *In what time a gross and petty average shall be demanded.*

82. *What proofs shall be made for a gross average.*

83. *What proofs shall be made for a petty average.*

84. *Upon renunciation by the assured,*